Preparation for Precalculus

Congratulations on your acceptance to the Governor's School of Southside Virginia (GSSV). I look forward to working with you as your mathematics instructor. I am confident that your experience at GSSV will be both challenging and rewarding.

As you know, students from different counties comprise the GSSV student body. Therefore, the mathematics background each student possesses can be quite different. In order to facilitate a smooth transition into Precalculus, it would be beneficial to have students enter the course having already mastered certain algebraic skills. Therefore, a review packet is enclosed for your use this summer to assist you in assessing your level of preparedness for the course and to provide exercises to help you "brush up" on skills that may be rusty. It addresses key concepts that are needed for success not only in mathematics but also in Chemistry. Should you have any questions or concerns or if you experience difficulties with the material and need assistance, please feel free to send me an email at ann.moore@southside.edu .

Enjoy your summer!

Order of Operations

It may seem that this is an elementary school topic; however, it is critical that students abide by the order of operations when simplifying algebraic expressions and solving complex algebraic equations. Many students remember the order of operations by using the pneumonic PEMDAS. This is fine if it is learned correctly! Specifically, multiplication and division are done in the same step according to the order in which they appear; order of appearance is determined by reading the problem from left to right as you would read a sentence in a book, for example. DO NOT multiply first and then go back to the beginning of the problem and divide next. The same holds true for addition and subtraction; these are done in the order they appear. Actually, division is another form of multiplication and subtraction is another form of addition. For example, $12 \div 2 = 12 \cdot \frac{1}{2}$ and $5 - 3 = 5 + -3$, so it makes sense that all multiplication/division or addition/subtraction would be performed in the same step. For upper level math courses, I prefer a more generalized approach to applying the order of operations.

1. Perform operations in grouping symbols. Grouping symbols are not limited to parentheses. Grouping symbols can be parentheses, brackets, braces, absolute value bars, radical signs, and the fraction bar.

2. Evaluate exponential terms. This is rather straightforward, but a surprising number of mistakes are made when completing this step. Close attention must be paid to how the term is written. $3x^2$ is not the same as $(3x)^2$. If x is 4, $3x^2 = 3 \cdot 4^2 = 48$ and $(3x)^2 = (3 \cdot 4)^2 = 12^2 = 144$. So, in algebraic terms, $ab^2 \neq (ab)^2$. This can be rather tricky when "a" represents "-1." $-3^2 = -9$; $(-3)^2 = 9$. Common mistakes are made when substituting for a variable. If $x = -4$, then $x^2 = (-4)^2 = 16$. If $x = 5$, then $-x^2 = -5^2 = -25$. Also, the calculator is not your friend in these situations. The calculator does exactly what you tell it to do!

3. Perform multiplication and division. As stated earlier, division is the same thing as multiplication. Dividing by 2 is the same operation as multiplying by one-half. Therefore, it makes sense that multiplication and division must be performed as one task. Order of appearance within the problem is extremely important. When given the problem $24 \div 3 \cdot 2$, the correct response is 16 since $24 \div 3 = 8 \; and \; 8 \cdot 2 = 16$. If a student multiplied first and then divided as implied by PEMDAS, an incorrect answer of 4 would result.

4. Perform addition and subtraction in the order of appearance. For a problem like $16 - 3 + 5$, the correct procedure would be to subtract first and then add giving a correct answer of 18. If the addition was performed first, an incorrect answer of 8 would be obtained.

Previously Observed Common Errors

1. Remember that the distributive property involves multiplication and takes precedence over addition or subtraction. In the algebraic expression $8+3(x-5)$, the multiplication involving 3 is completed before the addition of 8. Do not add first!
   $8+3(x-5)$ NOT $8+3(x-5)$
   $8+3x-15$ $11(x-5)$
   $3x-7$ $11x-55$

2. Grouping symbols must be removed before using a number contained in that grouping. In the previous example the 5 cannot be subtracted from 8 because the 5 is part of a grouped expression. Remove the parentheses by applying the distributive property, then combine like terms. Also, in a problem such as $\frac{(3x)^2}{27}$, the 3 can't cancel with the 27 because it is part of a grouping that is being squared. Raise to the second power first, then simplify by canceling. $\frac{(3x)^2}{27}=\frac{9x^2}{27}=\frac{x^2}{3}$.

3. Remember that fractions are a form of grouping; the numerator and denominator represent separate, individual values. The fraction $\frac{6x-3}{3}$ implies the grouping $(6x-3)\div 3$. The answer to $6x-3$ must be divided by 3. Many times students fail to recognize this grouping and immediately try to cancel the threes. $\frac{6x-3}{3}\neq 6x$. (Further the 3's don't cancel out as 3 divided by 3 is 1 giving an incorrect answer of 6x-1 and not 6x.) to simplify correctly, the 3 which is a common factor of both terms in the numerator should be factored out, then cancellation is allowed. $\frac{3(2x-1)}{3}=2x-1$. (In this case, 3 divided by 3 is 1 but the 1 is then multiplied by 2x-1 to give a correct solution.) Remember $\frac{3(2x-1)}{3}$ is equivalent to $\frac{3}{3}\cdot\frac{2x-1}{1}$; this form further validates the necessity to factor before cancelling.

4. Keep in mind that variables represent numbers and must be handled as numbers would be handled. Frequently, students try to simplify in a manner that is not consistent with the order of operations. $\sqrt{x^2+9}$ can't be simplified; however, some students will record that $\sqrt{x^2+9}=x+3$. This is not true! Students read $\sqrt{x^2+9}$ as $\sqrt{x^2+3^2}$, which is a correct interpretation, but the expression still can't be simplified. The square root is a grouping symbol, so the addition must be performed before a square root can be taken. This is easily proven using all numbers. Let's say that $x=4$. This problem becomes $\sqrt{16+9}$. Done correctly, we have $\sqrt{16+9}=\sqrt{25}=5$. If this problem is done without adhering to the order of operations, we would get $\sqrt{16}+\sqrt{9}=4+3=7$. This is not correct! We now have mathematical proof that we cannot separate (or individualize) the terms in an expression. Note: This is not to say that radical expressions can't be simplified. For example: $\sqrt{x^2+3x+9}=\sqrt{(x+3)^2}=|x+3|$. As you can see, this simplification uses inverse properties (squares and square roots).

5. The order of operations applies to algebraic expressions and equations. Given the expression $3(x-4)^2$, the order of operations require that the parentheses be squared first and then the 3 distributed. Unfortunately, too many students distribute first which is incorrect. Further, they square the parentheses incorrectly as x^2-16.

$$\begin{aligned} 3(x-4)^2 &= 3(x^2-8x+16) \\ &= 3x^2-24x+48 \end{aligned}$$

<u>Practice</u>

Without using a calculator, evaluate or simplify each expression.

1. $9+4(3^3-2^2)$
2. $2(4^2-2^3+3^0) \div \sqrt{25-16}$
3. $\{[8(5+2)-35 \div 7] \div 3\}^2$
4. $[(8^0+4) \cdot 3-1]^2$
5. $8(3^2-5)^{\frac{1}{2}}-[(16 \div 2^3)^2+5]$
6. $2+3(4-1)-\sqrt{9} \cdot 2$
7. $5(x-3)^2+2(x+1)$
8. $\frac{4x-12}{4}$
9. $15-3(x+2)^2+8x$
10. $\frac{(6x)^2}{4}$

Answers: https://tinyurl.com/y82hxyy6
Find more practice here.
http://www.mathworksheets4kids.com/order-of-operations/parenthesis-hard-2.pdf

Solving Equations for a Given Variable

When solving an equation for a given variable, the goal is to isolate that variable (the subject variable) on one side of the equation. We must use equation-solving techniques to accomplish this. Here is a link to a website that gives examples http://www.purplemath.com/modules/solvelit.htm and this is a link to a Khan Academy video https://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities/solving_for_variable/v/rearrange-formulas-to-isolate-specific-variables.

These links are to videos showing the rearranging process.

Transposition 1 - https://www.youtube.com/watch?v=0oq4arfe-SM
Transposition 2 - https://www.youtube.com/watch?v=yT_Z4OzPRaY
Transposition 3 - https://www.youtube.com/watch?v=J1NAcToXYjE
Transposition 4 - https://www.youtube.com/watch?v=HZ5kHTLeuMU

For the problems below, the equation is given first followed by the variable for which we are solving. (In number one, get b on a side by itself.) Remember to use inverse operations to "cancel out" numbers and variables on the side with the subject variable.

Try these.

1. $A=\frac{1}{2}bh;b$

2. $A=\frac{1}{2}(b_1+b_2)h;h$

3. $A=\frac{1}{2}(b_1+b_2)h;b_2$

4. $v=v_o+at;t$

5. $\frac{1}{4}x-\frac{1}{3}y-6=0;y$

6. $U_s=\frac{1}{2}kx^2;x$

7. $T_s=2\pi\sqrt{\frac{m}{k}};k$

8. $\Delta p=F\Delta t;F$

9. $W = F\Delta r\cos\theta; F$

10. $x = x_o + v_o t + \frac{1}{2}at^2; a$

11. $^{\circ}C = \frac{T_{^{\circ}F} - 32}{1.8}; T_{^{\circ}F}$

12. $n = \frac{m}{M}; M$

13. $x + 2q = \frac{c - 3x}{a}; a$

14. $S = \frac{b}{1 - v}; v$

15. $6m - 5n = 3m - 1; m$

16. $7a - 4b = ab - 2c; a$

17. $7a - 4b = ab - 2c; b$

18. $ax + (4a - b)(x + 3b) = 2x - a; x$

19. $at^2 - (3b + t)t - 4a + 7 = 0; a$

20. $c^2 - 6c = d + 14$; c

Factoring

When factoring the first thing you always look for is a greatest common factor.
Common factors are **factors** that the numbers being compared have in **common**. The **greatest common factor** is the largest **factor** that all the numbers being compared have in **common**. Thus, since there are not two or more numbers to compare, there are neither **common factors** nor a **greatest common factor**.

Kahn Academy Video https://tinyurl.com/yae35ecd
Try these examples

1. $3x^4 + 15x^3 + 21x^2$
2. $63b^2c^4 + 42b^3c^2$
3. $54z^3 + 18z + 36z^2$
4. $16y^3z^3 + 56y^4z$

Then, you will want to look at the following situations:

TRINOMIALS (Three Terms)

To factor polynomials where the leading coefficient is 1

1. Write the factors of the last number
2. Select the pair that adds to the middle number and multiplies to the last number

Try these

1. $x^2 + 8x + 15$

2. $x^2 - 4x - 5$

3. $x^2 - 14x + 45$

4. $x^2 - x - 90$

5. $x^2 - 9x + 81$

6. $x^2 - 7x - 44$

To factor trinomials where the leading coefficient is NOT one:

1. Multiply the first number by the last number
2. Write the factors of that number
3. Select the pair that adds to the middle number and multiplies to the sign of the last number
4. Create fractions with the choices and leading coefficient
5. Reduce the fraction
6. Use slip-n-slide to get rid of any fractions

Try These

1. $3x^2 + 7x + 2$

2. $5x^2 - 13x + 6$

3. $2x^2 + 5x - 3$

4. $3x^2 - 2x - 5$

5. $6x^2 - 17x + 12$

6. $8x^2 + 33x + 4$

BINOMIALS (Two terms)

Difference of Two Squares

Ask yourself three questions when you have a binomial to factor.

1. Can you take the square root of the first term?
2. Are the two terms connected with a minus sign?
3. Can you take the square root of the last term?

If you answer “YES” to all three questions, then you can factor using the difference of two squares.

Example: $16x^4 - 81$

Try these

1. $x^2 - 64$

2. $25 - (x + 5)^2$

3. $6x^2 - 54$

4. $12x^2 - 48$

5. $x^3 - 16x$

Difference of Two Cubes

When two terms in a binomial are cubed, you will need to follow the acronym SOFAS to factor. The two terms can be connected by a + or a –

For the first set of parenthesis take the cube root of each term and keep the sign, then for the second set of parenthesis follow SOFAS

S - Square the first term
O – Opposite sign
F – Front times back
A – Always add
S – Square the last term

Example

$$x^3 + 8 = (x + 2)(x^2 - 2x + 4)$$

Try these

1. $27 - x^3$

2. $u^3 + 27v^3$